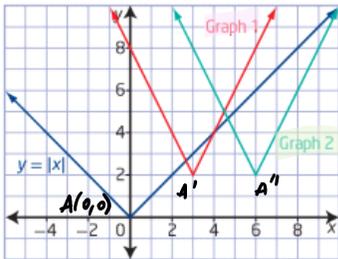


IT SHOULD NOT BE 3 BUT RATHER 6.

I THINK THIS IS A TYPO

13. Gil is asked to translate the graph of  $y = |x|$  according to the equation  $y = |2x - 6| + 2$ . He decides to do the horizontal translation of 3 units to the right first, then the stretch about the y-axis by a factor of  $\frac{1}{2}$ , and lastly the translation of 2 units up. This gives him Graph 1. To check his work, he decides to apply the horizontal stretch about the y-axis by a factor of  $\frac{1}{2}$  first, and then the horizontal translation of 6 units to the right and the vertical translation of 2 units up. This results in Graph 2.

- Explain why the two graphs are in different locations.
- How could Gil have rewritten the equation so that the order in which he did the transformations for Graph 2 resulted in the same position as Graph 1?



let's call it  $f(x) = |x|$   
 $g(x)$

then:

$$g(x) = |2x - 6| + 2$$

$$= f(2x - 6) + 2.$$

Graph 1 started w/ the H. shift, then followed w/ an H. stretch.

The formula that Graph 1 represents is

$$g_1(x) = f(2x - 3) + 2$$

6 typo! → see Desmos link at the bottom

H. stretch by a factor of  $\frac{1}{2}$  happens first

happens second b/c there are no brackets.

Graph 2 on the other hand represents: → third step

$$g_2(x) = f(2(x - 6)) + 2$$

first: H. stretch by a factor of  $\frac{1}{2}$  is applied first b/c we have brackets.

shift at 6 units to the Right happens second.

∴ not 3, it's

Let's compare the mappings ↳ b/c not 3, it's a typo!

Graph 1:  $g_1(x): (x, y) \rightarrow \left( \frac{(x+6)}{2}, y+2 \right)$

Graph 2:  $g_2(x): (x, y) \rightarrow \left( \frac{1}{2}x+6, y+2 \right)$

Let's see what happens in each case to a point from  $f(x)$ , for ex. the vertex  $A(0,0)$

$g_1(x): A(0,0) \rightarrow \left( \frac{0+6}{2}, 0+2 \right) = (3, 2) = A'$  ↳ the vertex as seen in Graph 1

$g_2(x): A(0,0) \rightarrow \frac{0}{2}+6, 0+2 = (6, 2) = A''$   
↳ the vertex as seen in Graph 2.

Graph 2 is incorrect, Graph 1 is correct, which is why we have different positions.

Graph 2 starts w/ the stretch: it would then have to use brackets:

$g(x) = |2x-6|+2 = f(2x-6)+2$  (\*)

↳  $f(2x-6)$  has no brackets inside the parameters, i.e.,  $2x-6$  should be  $2(x \dots)$

not these brackets!

these brackets!

factor 2 out of  $2x - 6 = 2(x - 3)$

make brackets so

u can see the stretch first, like so:

$f(|2x-6|) = f(2(x-3))$  (\*\*)

$$f\left(2\left(\frac{2x}{2} - \frac{6}{2}\right)\right) = f(2(x-3)) \quad (**)$$

H. Stretch first by a factor of  $\frac{1}{2}$

H. Shift 3 units to the Right

once you have used the brackets, it's no longer a H. Shift Right by 6 units, but rather by 3 units!

This is why Graph 2 has been moved 3 units too far to the Right by 3 units too far, at (6, 2) instead of having the vertex at (3, 2).

We know that the Graph 1 represents the function  $y = |2x - 6| + 2$  correctly, so the red diagram is correct.

**See a discussion about the typo here:**

[https://docs.google.com/presentation/d/e/2PACX-1vRjdkQd4G2LXT4fDgTm46iECgRW7ayqBmjGAdfi4-6uUEZDZ0kpo0701NyTDCt7eaNt6m99gyqyaLDe/pub?start=false&loop=false&delayms=3000&slide=id.g399e29b1fca\\_0\\_5](https://docs.google.com/presentation/d/e/2PACX-1vRjdkQd4G2LXT4fDgTm46iECgRW7ayqBmjGAdfi4-6uUEZDZ0kpo0701NyTDCt7eaNt6m99gyqyaLDe/pub?start=false&loop=false&delayms=3000&slide=id.g399e29b1fca_0_5)

The textbook answers the question so:

- it assumes Graph 1 is correct
- it assumes Graph 2 is incorrect
- the difference is due to the brackets and is based on the traditional form:

$$y = a f[b(x - h)] + k$$

13. a) The graphs are in two locations because the transformations performed to obtain Graph 2 do not match those in  $y = |2x - 6| + 2$ . Gil forgot to factor out the coefficient of the x-term, 2, from -6. The horizontal translation should have been 3 units right, not 6 units.
- b) He should have rewritten the function as  $y = |2(x - 3)| + 2$ .

b) When we blend  $(*)$  &  $(**)$  together, we get

$$g(x) = f(2x - 6) + 2 = f(2(x - 3)) + 2$$

He has to rewrite the equation as

$$f(2(x-3)) + 2 \quad \text{or} \quad y = |2(x-3)| + 2$$

so he can notice to shift by 3 to the Right instead of by 6.