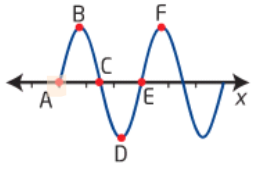
p 234 q12

12. The points indicated on the graph shown represent the x-intercepts and the maximum and minimum values.



a) Determine the coordinates of points B, C, D, and E if $y = 3 \sin 2x$ and A has coordinates $(0, 0)$.

b) Determine the coordinates of points C, D, E, and F if $y = 2 \cos x$ and B has coordinates $(0, 2)$.

c) Determine the coordinates of points B, C, D, and E if $y = \sin \frac{1}{2}x$ and A has coordinates $(-4\pi, 0)$.

$A(0,0)$

a) amplitude $= 3$

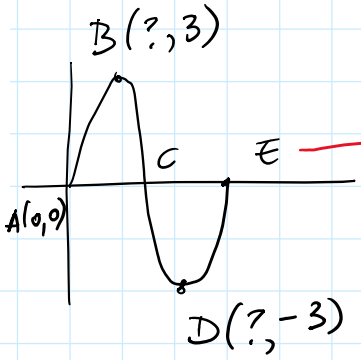
$\Downarrow$

$\max y = 3$

$\Downarrow$

$y_B = 3 \Rightarrow y_D = -3$

b/c it's the equivalent min.



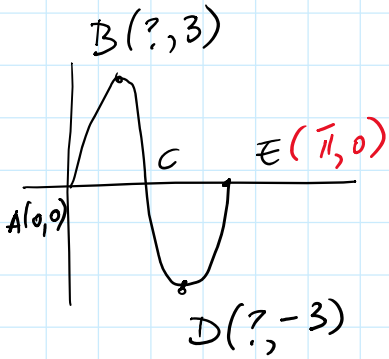
end of the period.

this is $3\sin 2x$

$P = \frac{\text{original period}}{2} = \frac{2\pi}{2} = \pi$

$\therefore x_E =$ end of period $= \pi$
and it's an x-intercept

$\Rightarrow E = (\pi, 0)$



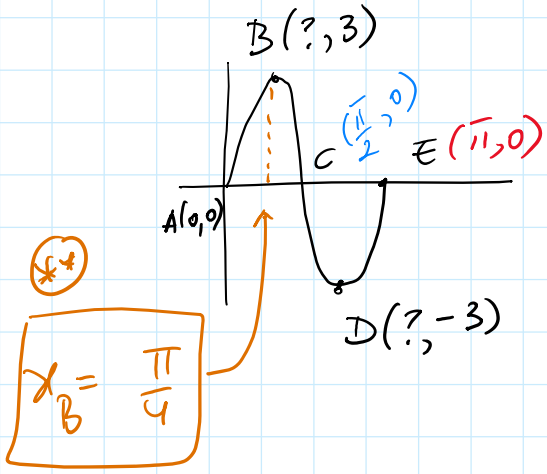
C is half way through

$\Downarrow$

$\frac{\pi}{2}$

it's also an x-intercept

$\Rightarrow C = (\frac{\pi}{2}, 0)$



$x_B = \frac{\pi}{4}$

$x_B = ?$ & $x_D = ?$

B is half way through $\frac{\pi}{2}$

so we could say $\frac{\frac{\pi}{2}}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$

we could also solve:

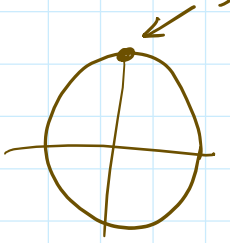
B: $y = 3\sin(2x)$

we know $y_B = 3$ $\Rightarrow$ $3 = 3\sin(2x)$ | solve for x

$\div 3$

$1 = 1 \cdot \sin 2x$

$\sin(2x) = 1$

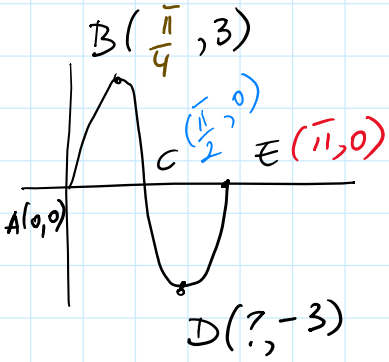


the angle is $\frac{\pi}{2}$ here

$\Rightarrow (2x) = \frac{\pi}{2}$ | $\div 2$

$x = \frac{\pi}{2} \cdot \frac{1}{2}$

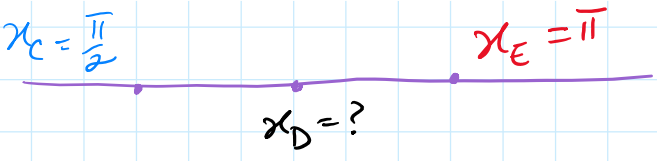
$x = \frac{\pi}{4}$ ← same as the other solution



similarly D is half way between C & E

$x_C = \frac{\pi}{2}$ $x_E = \pi$

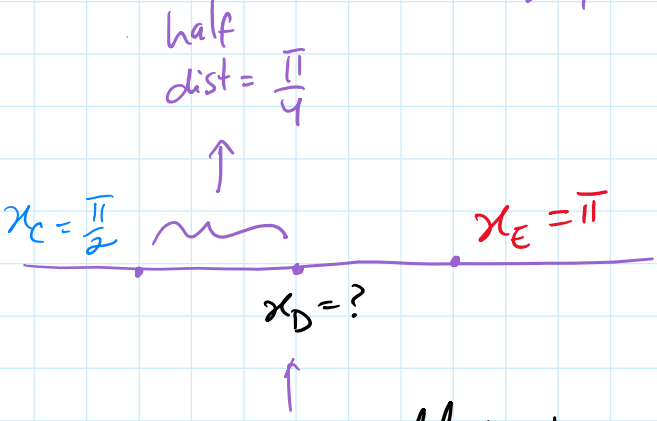
$D(?, -3)$



total distance $= \pi - \frac{\pi}{2} = \frac{\pi \cdot 2}{2} - \frac{\pi}{2} = \frac{2\pi - \pi}{2} = \frac{\pi}{2}$

$= \frac{\pi}{2}$

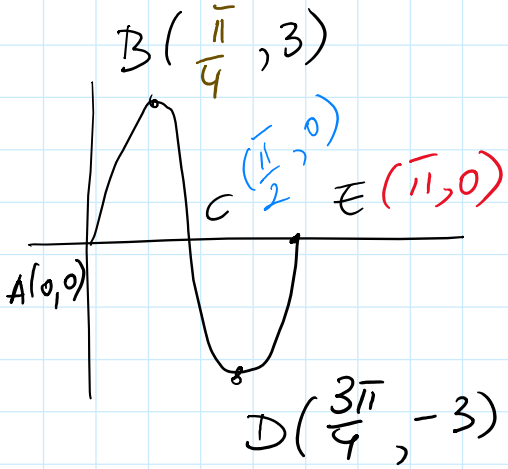
half the distance $= \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$



we add $\frac{\pi}{4}$ to x_c

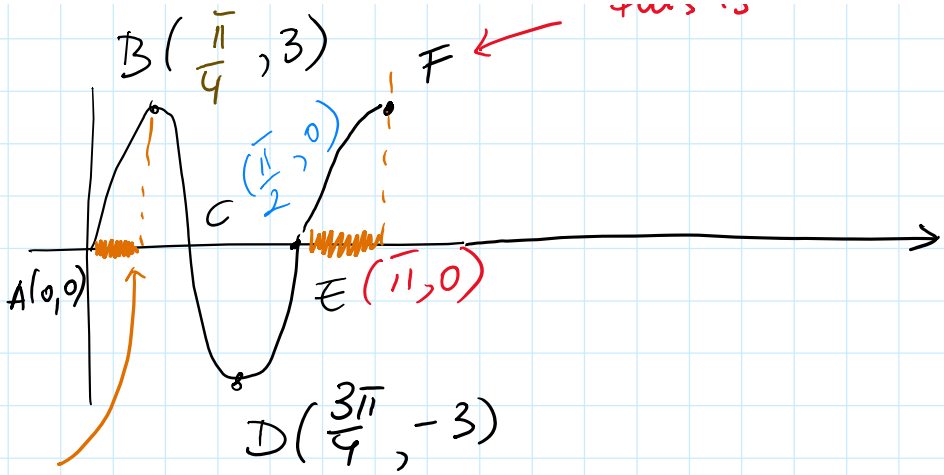
$$x_D = \frac{\pi}{2} + \frac{\pi}{4} = \frac{\pi \cdot 2}{2 \cdot 2} + \frac{\pi}{4} = \frac{2\pi + \pi}{4} = \frac{3\pi}{4}$$

$x_D = \frac{3\pi}{4}$



we hv all points except for F (oops, we didn't have to calc F)

$B(\frac{\pi}{4}, 3)$, F ← this is the next peak (= max)



$y_F = y_B$

$y_B = 3$

$y_F = 3$

$x_B = \frac{\pi}{4}$

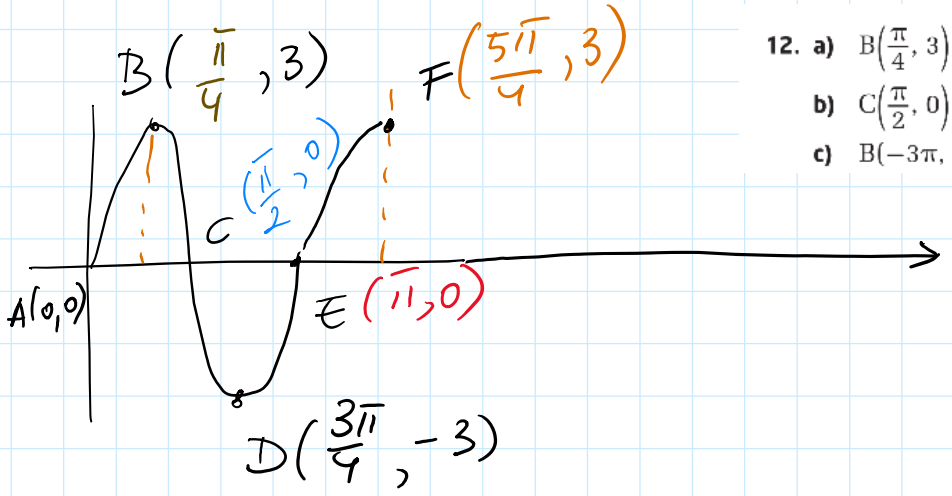
(see ✱✱) the distance between A & B $= \frac{\pi}{4}$

Symmetry & periodicity ensures that is the same distance btw E & F $\Rightarrow$

$$x_F = x_E + \frac{\pi}{4}$$

$$x_F = \pi + \frac{\pi}{4} = \frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$\Rightarrow F = \left(\frac{5\pi}{4}, 3\right)$$



12. a) $B\left(\frac{\pi}{4}, 3\right), C\left(\frac{\pi}{2}, 0\right), D\left(\frac{3\pi}{4}, -3\right), E(\pi, 0)$

b) $C\left(\frac{\pi}{2}, 0\right), D(\pi, -2), E\left(\frac{3\pi}{2}, 0\right), F(2\pi, 2)$

c) $B(-3\pi, 1), C(-2\pi, 0), D(-\pi, -1), E(0, 0)$