extra Q

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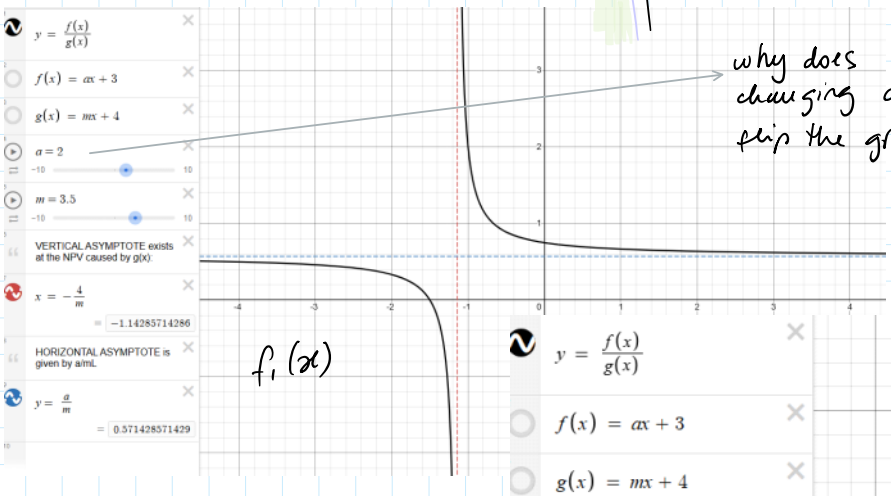
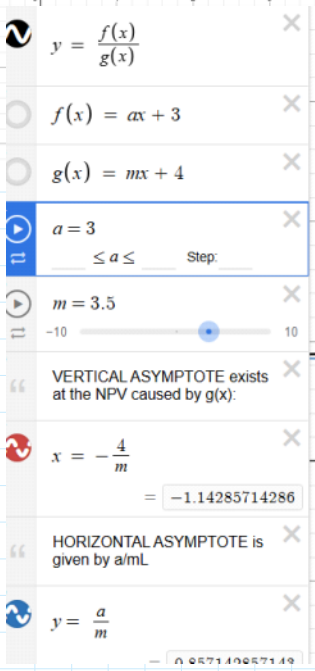
https://www.desmos.com/calculator/ioh0cp7url

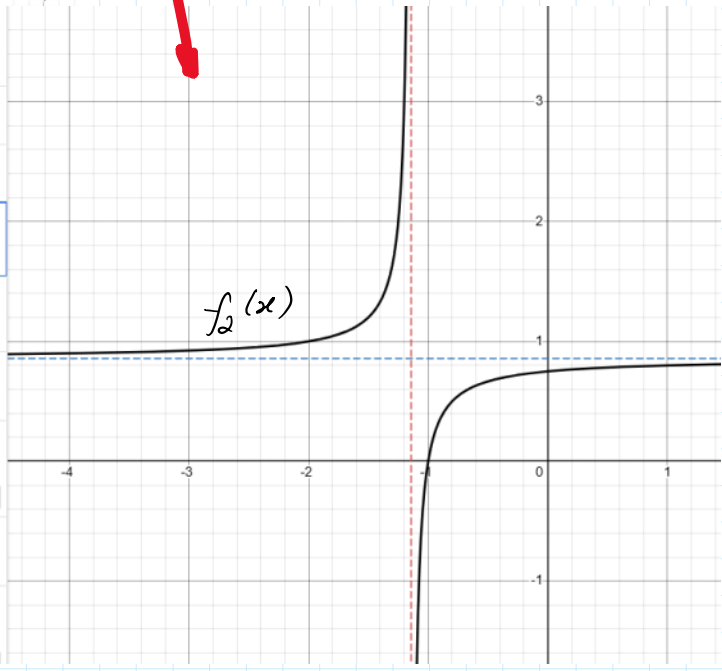


$f_1(x)$

why does changing $a=3$ flip the graph?

flipped (reflected)





Let's figure out the two functions

$$f_1(x) = \frac{ax+3}{mx+4}$$

(2 above $ax+3$; 3.5 below $mx+4$)

we evolve the form

$$y = \frac{a}{x-h} + k$$

& see what it tells us

$$f_1(x) = \frac{2x+3}{3.5x+4} \quad (3.5) \quad = \frac{\frac{2x+3}{3.5}}{\frac{3.5x+4}{3.5}} = \frac{\frac{2x}{3.5} + \frac{3}{3.5}}{x + \frac{4}{3.5}}$$

$$= \frac{\frac{2x}{3.5} + \frac{3}{3.5}}{x - \left(-\frac{4}{3.5}\right)}$$

$\Rightarrow h = -\frac{4}{3.5}$

$$\frac{\frac{2x}{3.5} + \frac{3}{3.5}}{x - \left(-\frac{4}{3.5}\right)} = \frac{a}{x - \left(-\frac{4}{3.5}\right)} + k$$

($x + \frac{4}{3.5}$)

(left side)

we want the same denominators

(right side)

we calculate the right side:

$$\text{"R"} = \frac{a + k\left(x + \frac{4}{3.5}\right)}{x - \left(-\frac{4}{3.5}\right)}$$

$$\text{"k"} = \frac{a + k\left(x + \frac{4}{3.5}\right)}{x - \left(-\frac{4}{3.5}\right)}$$

$$= \frac{kx + \left(a + \frac{4}{3.5}k\right)}{x - \left(-\frac{4}{3.5}\right)}$$

$$\frac{\frac{2x}{3.5} + \frac{3}{3.5}}{x - \left(-\frac{4}{3.5}\right)} = \frac{kx + \left(a + \frac{4}{3.5}k\right)}{x - \left(-\frac{4}{3.5}\right)}$$ eliminate the denominators

$$\frac{2x}{3.5} + \frac{3}{3.5} = kx + \left(a + \frac{4}{3.5}k\right)$$

coefficients constants.

$$\Rightarrow \begin{cases} \frac{2x}{3.5} = kx \\ \frac{3}{3.5} = a + \frac{4}{3.5}k \end{cases} \Rightarrow \begin{cases} \boxed{k = \frac{2}{3.5}} \\ \frac{3}{3.5} = a + \frac{4}{3.5}\left(\frac{2}{3.5}\right) \end{cases} \Rightarrow$$

$$\Rightarrow a = \frac{3}{3.5} - \frac{4(2)}{(3.5)(3.5)} \Rightarrow \boxed{a = \frac{10}{49}}$$

use calculator

The positive value of this a tells us the function $f_1(x)$ is no reflected compared to $\frac{1}{x}$. That is since $\frac{1}{x}$ is like so

So will $f_1(x)$ be in relation to its asymptotes:

What about the second function? $f_2(x)$ where its $a = 3$?

We want to calculate its $y = \frac{a}{x-h} + k$ form as well.

$$f_1(x) = \frac{ax + 3}{mx + 4} = \frac{3x + 3}{2x + 4} \overset{\div 3.5}{=} \frac{\frac{3x+3}{3.5}}{3.5x + 4} =$$

$$f_1(x) = \frac{ax+3}{mx+4} = \frac{3x+3}{3.5x+4} \overset{(3.5)}{=} \frac{\frac{3x+3}{3.5}}{\frac{3.5x+4}{3.5}} =$$

$$= \frac{\frac{3}{3.5}x + \frac{3}{3.5}}{x + \frac{4}{3.5}} = \frac{a}{x+\frac{4}{3.5}} + k$$

we want it to be equal to $\frac{a}{x-h} + k$

$x - \left(-\frac{4}{3.5}\right)$

$\hookrightarrow h = -\frac{4}{3.5}$, though we don't really care abt h.

$$\frac{\frac{3}{3.5}x + \frac{3}{3.5}}{x + \frac{4}{3.5}} = \frac{a}{x+\frac{4}{3.5}} + k \cdot \frac{\left(x+\frac{4}{3.5}\right)}{\left(x+\frac{4}{3.5}\right)}$$

← eliminate the denominators

numerators only:

$$\frac{3}{3.5}x + \frac{3}{3.5} = a + k\left(x+\frac{4}{3.5}\right)$$

$$\frac{3}{3.5}x + \frac{3}{3.5} = kx + \left(a + \frac{4}{3.5}k\right)$$

$k = \frac{3}{3.5}$

$$\frac{3}{3.5} = a + \frac{4}{3.5}\left(\frac{3}{3.5}\right)$$

$$\frac{3}{3.5} - \frac{4(3)}{(3.5)(3.5)} = a$$

$$\Rightarrow a = \frac{3(3.5) - 12}{(3.5)(3.5)}$$

$$a = -\frac{6}{49}$$

← in this case the a is negative

⇓

a vertical reflection into the x-axis must be present.

You can verify both $f_1(x)$ & $f_2(x)$ calculations

here: https://www.desmos.com/calculator/qmpcm3lgsh

In conclusion, the $y = \frac{a}{x-h} + k$ form reveals all the details necessary to plot the function correctly.